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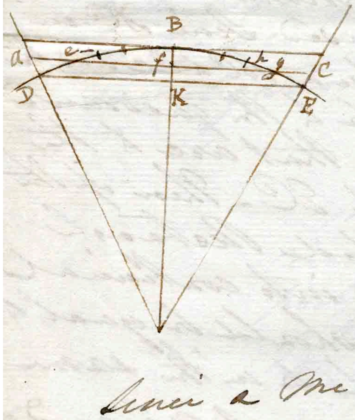
To Thomas Young Esquire M.D. F.L.S. &c &c
Secretary of the Royal Society
For: Correspondence
London

Sir

I have the honor to submit for the consideration of the Royal Society the following demonstration connected with the Theory of the Quadrature of the Circle founded on the Doctrine of proportionals, superseding the principle heretofore in practice of ascertaining the area of figures by subdividing the circumference into a specific number of sides, and taking the mean between polygons inscribed within and described about the Circle — that principle of computation being mathematically imperfect.—



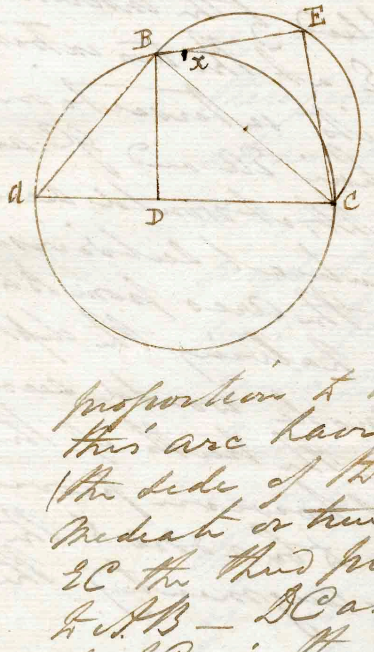
I respectfully submit that in taking the mean of the two polygons a small portion of the subdivided arch is clearly lost as the Mediate line AC in Figure 1. (of the mediate polygon) cuts off a larger portion of the segment DBE than is supplied by the intersticial additions AeD. CgE at the two extremes of the arc beyond its intersection with the mediate line EA and GC: and since a mediate polygon is sought, it would seem more consistent with mathematical subdivision to find a medium, by halving the arcs from B to E and D to B at i and h, than by halving the depth of the segment BK at f; this mode would seem to be an approximation nearer the truth of the actual result; the argument however proves that a fraction still remains to be added to the asserted proportion of 3.14159 &c augmenting the area of the Circle accordingly

The object hence arises to trace some proportional affording by demonstration the result of this approximation—

The remarkable properties of the rectangle inscribed within a semicircle upon

which

which rectangle all the proportions of circles are positively
dependent appear to afford ~~the means of demonstrating~~
H ~~the true proportional for the quadrant line~~ – Circles
being in due proportion to the squares of their
respective diameters, their areas being as the squares
of the sides of the Rectangle supposing the two sides
subtending the right angle to be made the diameters
of other circles, the sum of the Areas of circles formed
upon these subtending sides giving also the area of
the greater circle formed upon the base of the semi-
-circle, throughout the whole series of its periphery
i.e. supposing the right angle to be transferred to any
part of the arc of the semicircle appear to afford the
means of demonstrating the true proportional for the
quadrant line – Then by assuming one of the sides of
the inscribed Rectangle as the Measure of the Quadrant
arc expressing the proportion of the Circle to the Square
of its own diameter, this arc being found to be in the
same ratio, a mediate proportional will be had in the
dimensions of the arc thus reduced to a straight line
and a semicircle upon its base BC (Figure 2) with



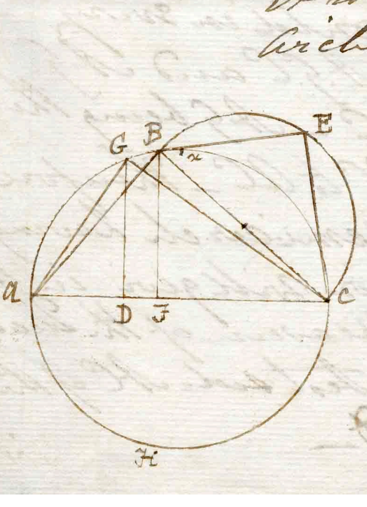
given a third proportional in the line
EC equal to the versed Sine DC in
third proportion also as BC to AC and the
diameter AC will be to the quadrant
arc or line BC as that arc to the
third proportional EC – Then if the
sides of the inscribed Rectangle
ABC were reduced into continual
proportion – AB should be equal to
EC – i.e. AB should be in the same
proportion to BC as EC to the Quadrant arc and if
this arc have been finally expressed by the line BC
(the side of the Rectangle) that line should be in the
mediate or true proportion between AC and EC and
EC the third proportional would be equal to DC or
to AB – DC and AB being also third proportionals
to AC i.e. they are to BC as BC to AC – for by construction
the sides of the Rectangle ABC are in continual
proportion but the given ratio of the quadrant arc
BC namely 7.8539 &c. furnishes a third proportional
of 6.1685316 whilst DC or AB the versed line and
side

side of the Rectangle is each equal to 6.18034 &c as the third proportion in the inscribed Rectangle ABC – Now it is evident that if BC be the common measure or second proportion regulating the third proportions EC and AB the mediate proportion BC of the Rectangle must be equal to BC the quadrant arc as the mean proportional – Since the assumed equality of the line BC with the arc xc admits of the reduction of the three proportions into a rectangle inscribed within and upon the base of the semicircle which if they did not no rectangle so inscribed could be reduced to a continual proportion in its sides but that is impossible as the case does admit of such reduction, for the mediate proportional BC of the Rectangle is the true expression of the quadrant arc and consequently the proportional number 7.8539 &c is not the final expression of the quadrant arc of the circle –

Consistently with this argument I have the honor to submit with all due deference the following proposition and elucidation of the mean proportional number which is the final expression of the quadrant arc and to solicit the favor of the Royal Society's consideration of the same, being influenced with a faithful conviction ~~that the~~ solution will in their estimation be deemed worthy of Mathematical investigation –

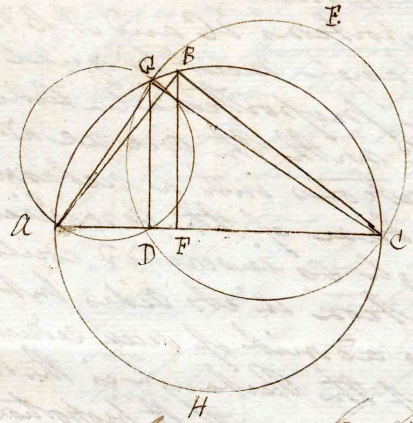
Proposition

That a Rectangle inscribed within a semi circle having its sides in continual pro= portion i.e. as the greatest side (being the base of the semicircle) is to the second side so is the second to the third or least side; the second side will be the true or final expression of the Arc of the Quadrant of a circle of which the base of the semicircle is the diameter i.e. it will give the true proportion of the circle to the square of its diameter –



Let ABC be any semicircle of which AC is the base the inscribed Rectangle AGC having its sides AC, GC, AG in continual proportion i.e. as AC is to GC so is GC to AG – the side GC will be the true expression of the quadrant Arc xC or the proportion of the

Area



area of the Circle A.B.C.H. to the
square of the diameter (A.C.) —
Upon the line BC. the measure
of the quadrant arc draw the semi=
=circle BEC and make EC a third pro=
portional to AC or to BC as BC is to AC.
Since EC is a third proportional the
lines AC, BC, EC may be reduced
into a Rectangle inscribed within
a semicircle of which AC is the base but this is
the identical Rectangle sought — Consequently BC
the measure of the Quadrant arc is equal to GC the
mean proportional in the Rectangle and the third
proportional EC of the quadrant arc is equal to
the side AG of the inscribed Rectangle and BC
being the supposed true expression of the mean pro=
=portional or quadrant arc between AC and EC —
GC the mean proportional in the inscribed rectangle
being equal to it is the true expression of the
quadrant arc — and if BC be not equal to GC —
BC is not a true proportional nor the final
expression of the arc —
Draw DG and BF each perpen=
=dicular to AC — EC being a third proportional
and equal to FC — FC is also a third proportional
for DC is to GC as GC to AC — FC and DC are
therefore equal to each other, but DC is greater
than FC therefore FC is not the third proportional
therefore BC is not the mean proportional which
it was said to be by construction: Therefore GC
is the true mean proportional and if BC were
true also the Triangle ABC would in every
respect be equal to the Triangle AGC and AG
would be equal to EC and to FC — AG being the
third proportional or to GC as GC to AC — Therefore
a Rectangle inscribed within a semicircle having
its sides in continual proportion will give in the
mean proportional GC the true measure of the Quadrant
arc of the Circle of which the greatest side AC of the
Rectangle is the diameter: 128 D —
Cor:

Corollary – The sum of the Areas of Circles of which GC and AG are diameters will be equal to the area of the circle of which AC is diameter and moreover the circle of which AG is diameter will be to the circle of which GC is diameter as this further circle is to the circle of which AC is diameter

Arithmetical Formula

The Diameter AC = 10 – The Quadrant or arc xc gives = 7.853981648.
Then the third proportional EC to xc will be found as follows–

as 10 : 7.8539 :: 7.8539 : 6.1685 = EC

The Triangle A.G.C being in continual proportion gives the following numbers–

AC = 10.–
GC = 7.8615 &c
AG = 6.18034 nearly = DC

EC should be equal to DC but EC is = 6.1685
and DC is = 6.18034
is less by — 0.01184

Then as DC is to difference so is Semi-diam to the difference lost by the old scale on the radius

or as 6.18034 : 0.01184 :: 5.– : 0.00906 nearly

or in the semi-circumference 0.00906 × 31.446 = .284909 &c

Proof

The sides of the Rectangle A.G.C and their squares are as follow in continual Proportion giving the like proportions in circles of which they are diameters and their eventual agreement in continual proportion

AC = 10 — AC^2 = 100.
GC = 7.8615 &c — GC^2 = 61.8034 nearly
AG = 6.18034 — AG^2 = 38.1966 &c
$GC^2 + AG^2$ = 100.–

Again

GD = 4.858680 &c GD^2 = 23.606795 nearly
DC = 6.18034 nearly DC^2 = 38.19661 ditto
$GD^2 + DC^2$ = 61.8034 nearly

Lastly

GD = 4.858680 &c — GD^2 = 23.60679 &c
AD = 3.81966 &c — AD^2 = 14.58981 nearly
$GD^2 + AD^2$ = 38.19660

And if DJ be drawn perpendicular to GC the further proportions of sides will be as follow

GJ = 3.00282 &c — GJ^2 = 9.01700 &c
AD = 3.81966 &c — AD^2 = 14.58981 &c
$GJ^2 + AD^2$ = 23.60681 &c

On

On the foregoing data and proportionals the greater and Subsidiary Circles of which the several sides of the Rectangle and its components are diameters will give the following areas of those Circles found by multiplying half the Circumference into half their respective diameters these areas being in continual proportion as the sides of the Rectangle viz.

First — Diameter = 10. ——
Quadrant = 7.861507 &c
Circumference = 31.446029 &c
Area of the greater circle of which AC is Diameter = 78.61507 &c

Secondly — Diameter = 7.861507 &c
Quadrant = 6.18034 &c
Circum: = 24.72132 nearly
Area of the Second Circle of which GC is Diameter = 48.586805 &c

Thirdly — Diam: = 6.18034 &c
Quad.t = 4.858608 &c
Circum. = 19.4347 nearly
Area of the third Circle of which AG is diam: 30.028268 &c

In continual proportion

The sum of the two last Areas being equal to the First Area — 78.61507 &c

The three Areas are in continual proportion and the proportions of Areas to the squares of the diameters being the three sides of the Rectangle are as follow

as AC^2 : GC^2 :: Area AGCH : Area GECD
100 : 61.8034 :: 78.61507 : 48.58608 &c
as AC^2 : GC^2 :: Area GECD : Area AGD
100 : 61.8034 :: 48.58608 : 30.028268 &c

And

78.61507 : 48.58608 :: 48.58608 : 30.028268 &c
48.58608 + 30.02826 = 78.61507 &c Area as before

The Areas and Quadratures are as follow

Area AGCH = 78.61507 — Quadrature 7.8615 &c = GC
Area GECD = 48.5860 — Do —— 6.18034 = AG
Area AGD = 30.02826 — Do —— 4.858608 = GD

The Agreement between the Areas of these Circles and the sides and Components of the Rectangle is as follows

The side GC = $\frac{\text{Area AGCH}}{10}$
The side GD = $\frac{\text{area GECD}}{10}$
The side GI = $\frac{\text{area AGD}}{10}$

The above agreement in the case adduced with the diameter 10 proves that the Areas of Circles are in the decimal ratio to the sides and components of an inscribed moderate Rectangle. —

The

The difference between the former and present theories will be as follows—

Circumference ϕ	Former Theory ——	31.415926 &c
ϕ	Present Theory ——	31.446029 &c
	+	0.030103

Which difference distributed into a polygon of 1536 sides on which subdivision the former scale or theory is founded will give a quotient to each side of such polygon a portion of its segment cut off equal to .00002 nearly — to be added to the result of the old scale or lost in the area of each segment + 0.002008 &c—

This difference is in favor of the theory advanced that in taking the mean of the two polygons a certain fraction is lost as before stated; this fraction would be less by halving the subdivision of the arc as is before stated.

By the present solution the loss upon each subdivision of 1536 parts of the circumference is found at 0.00002 nearly to be added to the result of the former theory or computation —

Finally it may be observed that the square of the Quadrant arc when considered as the extreme power of the Square figure gives a clear difference with the computed Area of the circle, of 16.854 by the old scale or by the scale now had 16.811 — this difference is found to lessen as the sides of the square figure are multiplied and can only then disappear in toto when the square figure assumes the true character of the circular curve; that is in a pentagon it would be reduced as 5 to 4 — in a hexagon as 6 to 4 — in a polygon of 1536 sides as 1536 to 4 and in a square figure of n sides as n to 4 — In a figure of 1536 sides the difference of the square and Circular Areas is reduced accordingly to +.0043 — (Now the difference between the two found Areas — 78.539 &c and 78.61507 &c — is .0753 which divided by 31.446 the circumference gives .002008; — this item multiplied by .000196 the part of each subdivision of 1536) or in a figure of 1000000 sides to +.0000168 &c (or with a subdivision of 2194 sides we have +.030103) and by further subdivision the decimal only becoming more distant without totally disappearing so that the fraction found by comparing the former and proposed scales becomes the more important as an ultimate agreement arising from a demonstration founded on proportionals. —

In the hope and solicitude that the Royal Society will be graciously pleased to confer upon me a recommendation to His Majesty's Government

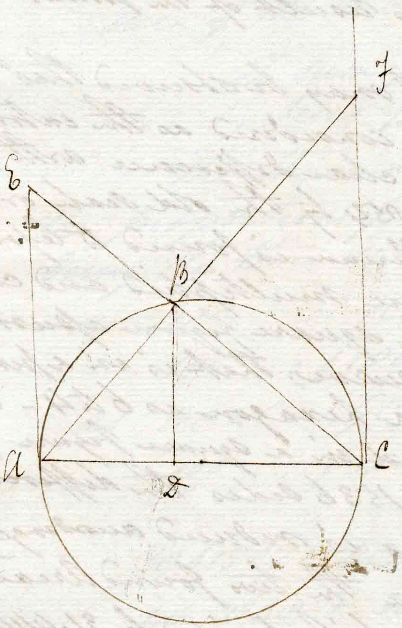
in

in the event of the present solution meeting with their approbation —

I have the honor to be
Sir
With the highest respect and consideration
Your most obedient and faithful, obliged Servant
Charles Hudson

Calcutta in Bengal
the 20th of January 1825
No 145 Bow Bazar Street
or at the Assist Genl's Office
(on duplicate)

Postscriptum — Sir — Availing myself of the present opportunity I beg respectfully to subjoin a further elucidation of the agreement of the quadrant arc with the proportional mean treated of in the foregoing proposition



Let BC (the measure of the quadrant arc) be drawn to E and AB to F and CF and EA be also drawn — the Right angled Triangles EBA — ABC and CBF will have their angles similar and their sides in respective proportion — The side FB is to BC as BC to AB as AB to BE (BE equal to BD) and CD is to BD and BD to DA — The triangle ABC will be a mean proportional between EBA and CBF — BC the quadrant arc is a mean proportional between BF and BA also between AC and DC — Therefore the quadrant arc is a mean proportional between AC and ~~BC~~ AB the other two sides of a Right angled Triangle inscribed within a semicircle as before —

Your most obedient and faithful humble Servant
(signed) Charles Hudson
2nd April 1825

(True Copy)
Charles Hudson